

CHARACTERISTICS OF THERMAL RADIATION IN  
ISOTHERMAL AXISYMMETRIC CAVITIES

V. P. Simonov

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Radiative heat transfer is investigated in an isothermal axisymmetric cavity with a spherical end and a lateral surface which is the frustum of a cone. Figure 1 shows a schematic diagram of the cavity. In deriving the system of integral equations describing the radiative energy transfer in the cavity it was assumed that the radiation and reflection processes are diffuse and that the emissivity  $\epsilon$  and the temperature  $T$  of the cavity walls are constant.

The solutions of the system of equations are universal and are obtained by a numerical method described in [1]. In the investigation it was assumed that  $\alpha_1 = \alpha_2 = \alpha$ , and cavities were considered with  $\alpha = -20^\circ, 0^\circ, 20^\circ$ ;  $L/R = 0.5, 1.0, 2.0$ ;  $\epsilon = 0.3, 0.5, 0.7, 0.9$ .

In addition to the local values of the apparent emissivity  $\epsilon_a = B/\delta T^4$  for cavities having  $\epsilon = 0.3$ , the effective emissivity  $\epsilon_{\text{eff}} = Q/\pi R^{*2} \delta T^4$ , which determines the effect of the cavity, is given as a function of the apex angle of the cavity  $\alpha$  for various  $\epsilon$  and  $L/R$  and as a function of the length of the conical portion of the cavity  $L/R$  for various  $\epsilon$  and  $\alpha$ . It was shown that even relatively short cavities ( $L/R = 2.0$ ) have practically a maximum effect from the point of view of the energy loss of the heated surface. For  $\epsilon = 0.3$  the value of  $\epsilon_{\text{eff}}$  does not differ by more than 10% from the corresponding value for a cavity having the shape of a pointed cone ( $L/R \rightarrow \infty$ ). For larger values of  $\epsilon$  this deviation becomes less than 1-2%. The results of the investigation described in the article permit rapid estimates of the effect of a cavity. This is important in calculating the energy loss of a rough heated surface.

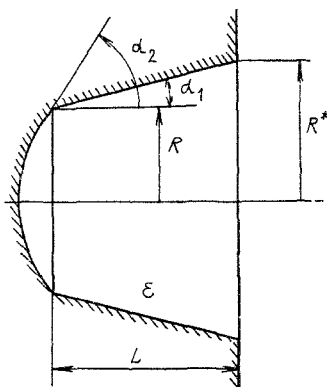


Fig. 1. Schematic diagram of cavity.

NOTATION

- $\epsilon$  is the emissivity of cavity walls;
- $T$  is the temperature of cavity walls;
- $B$  is the flux of effective radiation;
- $\epsilon_a$  is the local apparent emissivity;

†All-Union Institute of Scientific and Technical Information.

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- $\epsilon_{\text{eff}}$  is the effective emissivity;  
 $\sigma$  is the Stefan-Boltzmann constant;  
 $Q$  is the total heat flux radiated by open end of cavity;  
 $\alpha$  is the apex angle of cavity;  
 $L$  is the length of conical portion;  
 $R$  is the radius of base of frustum of cone;  
 $R^*$  is the radius of open end of cavity.

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#### MASS TRANSFER IN A THREE-PHASE SYSTEM

G. A. Aksel'rud and A. I. Dubynin

UDC 66.015.23

Heterogeneous reactions in the diffusion region [1] which are accompanied by the release of products, one of which dissolves in a liquid and the other of which is released as a gas, are discussed.

If the liquid is quiescent, the quantity  $q_g/\rho_g w_c$  plays the decisive part; here  $q_g/\rho_g$  is the rate of gas formation, while  $w_c$  is the speed of natural convection. If  $q_g/\rho_g w_c$  is sufficiently small, the effects of bubble production are negligible, and the mass-release coefficient  $k_R$  can be derived from standard relations for natural convection (in the dissolution of a solid in a liquid without the formation of gas). As  $q_g/\rho_g w_c$  increases, it plays an increasing part in determining the mass-transfer rate; conditions are set up in which the gas comes to dominate the process.

The following equation has been derived from a survey of the experimental data [1] on mass-transfer kinetics for some metals and carbonates reacting with acids in quiescent liquids:

$$\frac{k_R}{k_R^*} = 75 \left[ \frac{q_g}{\rho_g w_c} \right]^{0.65} \quad (1)$$

If the motion of the liquid with respect to the dissolving solids is of forced type, the mass-transfer rate is governed by the perturbations in the diffusion layer consequent on the gas release, as well as by the perturbations due to the force-convection. If the reagent concentration is small, the mass-release coefficient approaches the values calculated from the standard formula for convective mass transfer:

$$\text{Nu} = 0.8 \sqrt[3]{\text{Pr}} \sqrt{\text{Re}} \quad (2)$$

In the case of bubble release, the laws of mass transfer may be described by an equation of the type

$$\frac{k_R - k_R^{**}}{k_R^*} = f(\text{Re}), \quad (3)$$

which for these conditions reduces to the form

$$\frac{k_R - k_R^{**}}{k_R^*} = 0.013 \text{Re}^{0.47} \quad (4)$$

#### NOTATION

- $q$  is the mass flow density;  
 $\rho$  is the density;  
 $k$  is the mass-release coefficient.

## Indices

- g is the gas;  
R is the reagent (acid);  
r is the reaction products (soluble solids);  
\* is the parameter determined for natural convection;  
\*\* is the parameter determined for a quiescent liquid.

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## CALCULATING HEAT EXCHANGERS OF THE CROSS-FLOW TYPE WITH A FLUIDIZED BED OF MATERIAL

Yu. P. Nekhlebaev, I. A. Dorogoi,  
D. T. Bondarenko, and V. V. Konev

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The organization of heat and mass transfer in a heat-exchanger with a cross-current (HCC) of the fluidizing medium and material (FMM) is more efficient than in one of the mixing type. A computing relation was derived in [1] for determining the current temperature of the FMM along the HCC for a constant temperature of the fluidizing medium under the lattice of the HCC in the absence of heat absorption in the bed of material.

When heat absorption takes place in the bed as a result of the occurrence of endothermic reactions (the absorption being identical all along the apparatus), the following equations are obtained for determining the average temperature of the fluidizing medium and material at the exit from the HCC:

$$\bar{t}_2 = t_1 - \frac{q}{c_T W} + \left( \vartheta_1 - t_1 + \frac{q}{c_T W} \right) \frac{1 - \exp(-W)}{W},$$
$$\vartheta_2 = t_1 - \frac{q}{c_T W} + \left( \vartheta_1 - t_1 + \frac{q}{c_T W} \right) \exp(-W).$$

In this paper the combined operation of two HCC is considered in the case of complete mixing of the fluidizing medium under the lattice of the first HCC and also in the absence of mixing. Equations are derived for determining the current temperature of the FMM along the first HCC.

The average temperature of the FMM is calculated after traversing two preheating zones of a fluidizing-bed lime kiln. The results are explained on the basis of a redistribution of the temperature of the media taking part in the heat transfer. Recommendations are made as to a method of allowing for the actual distribution of the fluidizing medium under the lattice of the HCC for various degrees of heat absorption in the bed of material.

## NOTATION

- $c_T$  is the specific heat of fluidizing medium;  
 $q$  is the specific flow of heat to the endothermic reaction taking place in the bed of material;  
 $t_1$  is the temperature of the fluidizing medium at the HCC entrance (inlet);  
 $t_2$  is the average temperature of the fluidizing medium at the HCC exit (outlet);  
 $W$  is the ratio of the water equivalents of the fluidizing medium and the material;  
 $\vartheta_1, \vartheta_2$  are the temperatures of the material at the HCC inlet and outlet, respectively.

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## KINETICS OF DIELECTRIC DRYING OF CARDBOARD AND PAPER

V. M. Minakovskii

UDC 676.2.052.7

Results are given on the drying of specimens of cardboard (grade 350) and wrapping paper [GOST (All-Union State Standard) 7438-55] of size 215 × 130 mm heated by currents at frequencies of 64.7-66.3 MHz in a tubular capacitor connected to a high-frequency oscillator based on a GU-48 triode.

The curves clearly reveal the initial heating period (up to 18% of the total drying period), the period of constant drying rate, and the period of falling drying rate. Rapid and clearly visible steam release from the specimen starts at the middle of the heating period and continues not only in period I of the drying, but also, in part, in period II. The structure of the cardboard or paper favors the production of steam within the volume. When the field is switched off, the steam release ceases almost instantly, which indicates that the excess pressure relaxes completely before the specimen has cooled appreciably.

The drying rate  $N$  during period I for specimens of cardboard increased linearly with the initial water content  $W_0$  from  $W_{CR,1}$  up to  $W_0 \approx 190\%$ ; the rate of increase in  $N$  was independent of the field strength  $E$  in the material. Also,  $N$  was independent of  $W_0$  for  $W_0 \geq 190\%$ . Under comparable conditions,  $N$  for paper was larger by factors of 5, 5.65, and 6 than  $N$  for cardboard, respectively, with plate voltages  $U = 2.04, 2.33, \text{ and } 2.62$  kV, while the drying rate in period I, as referred to the total geometrical surface area of the specimen, was higher for the cardboard and varied from  $3.11 \text{ kg/m}^2 \cdot \text{h}$  (paper;  $W_0 = 159\%$ ;  $U = 2.04$  kV) to  $13.42 \text{ kg/m}^2 \cdot \text{h}$  (cardboard;  $W_0 = 246\%$ ;  $U = 2.62$  kV).

The drying curves for period II are divided into two parts by the point  $W_{CR,2}$ ; for cardboard with  $W_0 > W_{CR,1}$ , the values were  $W_{CR,1} \approx 60\%$  and  $W_{CR,2} \approx 30-32\%$ , these being independent of  $W_0$  and of the mode of drying. If  $W_0$  was close to  $W_{CR,1}$  or less than it,  $W_{CR,2}$  decreased as  $W_0$  decreased (to  $W_{CR,2} \approx 18\%$  for  $W_0 = 69\%$ ), while remaining independent of the mode of drying. In the case of paper,  $W_{CR,1} \approx 96\%$  for  $W_0 = 159\%$ , being independent of the mode of drying, while  $W_{CR,2}$  increased linearly from 46.5 to 58% as  $N$  increased from 5.3 to  $12\% \text{ sec}^{-1}$ .

The drying coefficients  $K_1$  and  $K_2$  increase linearly with  $N$ ; the relative drying coefficients  $\kappa_1$  and  $\kappa_2$  are independent of the mode of drying for  $W_0 > W_{CR,1}$  but increase as  $W_0$  falls. In the case of cardboard,  $\kappa_2$  for  $W_0 < 69\%$  is dependent on the mode of drying, and it decreases as  $N$  increases. Drying curves calculated from  $K_1$  and  $K_2$  agree well with the recorded curves.

Specimens uniform in water content showed no buckling throughout the drying, which is a difference from infrared and nozzle forms of drying.

The relationship of  $N\tau$  to  $N$  and the generalized drying curves as  $W$  in relation to  $N\tau$  confirm Krasnikov's hypothesis on the dielectric drying of cardboard in periods I and both parts of II, as well as for paper in period I and the first part of period II.

The  $N = N(U^2)$  curves indicate that the effects of  $E$  on the water-loss coefficients for cardboard and paper increase as  $W_0$  falls.

If a specimen uneven in water content is dried, the unevenness in the water distribution becomes less during period II, especially during the second part of this period. Formulas are given for estimating the equalization of the water content.

Information on the structure and electrophysical parameters of cardboard and paper can be used with concepts on the water-transport mechanism to explain all the observed trends in the dielectric drying of cardboard and paper.

# CURRENT-CARRYING CAPACITY OF A CRYOGENIC LEAD

Yu. L. Buyanov and A. B. Fradkov

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Measurements have been made on various types of copper lead used for supplying current to superconducting magnets.

If the lead has to carry a current that varies with the working conditions (the maximum current is supplied for a time small compared with the total working time) or if a superconducting magnet is operated in the frozen-field mode, it is advantageous to reduce the current-carrying cross section, since this reduces the heat leak to the liquid-helium cryostat when the current is small or zero.

Reducing the cross section increases the Joule heating and thus the temperature rise in the overload state; lead design thus requires a knowledge of the temperature distribution, in particular, the maximum temperature  $T_{\text{hot}} = \Psi(I)$ , which determines the permissible current.

The method of [1] for calculating the temperature distribution for  $I > I_{300}$  on the basis of ideal heat transfer between the lead and the surrounding gas appeared inapplicable for this type of lead on account of the different trends in the temperatures of lead and gas when the current is high.

The main interest attaches to the temperature in the part of the lead that approaches the melting point  $T_{\text{mp}}$  most rapidly. Experiments indicate that the temperature rises very rapidly with the current (an increase in current of only 20-25% over  $I_{300}$  may cause the lead to fail). It has also been found that heat flows from the hottest part to the outer section of the equipment, as well as into the cryostat, if that part has a temperature above room temperature, and then the hottest part is not at the input flange of the cryostat but at a distance of about 0.2 of the total lead length, which agrees with data for tungsten and molybdenum leads [2].

It has also been found that for copper leads, which have  $T_{\text{mp}} = 1356^\circ\text{K}$ , the observed  $T_{\text{hot}} = \Psi(I)$  are fitted closely by

$$T_{\text{hot}} = 300 \exp [1.51 (I - I_{300}) / (I_{\text{mp}} - I_{300})]. \quad (1)$$

The following empirical relation applies for the actual cooling conditions, which define the limiting current at which the lead melts:

$$I_{\text{mp}} = 100 S^{0.6}. \quad (2)$$

A theoretical relationship has also been derived for the permissible current in the general case.

The total amounts of heat introduced by the leads working under overload conditions have been calculated for the magnet supply time and for the working time and compared with the heat influx due to leakage along the leads, the latter as optimized for the maximum working current; it is found that the use of overloaded leads is advantageous for  $\tau_{\text{wor}} / \tau_{\text{sup}} \gtrsim 8$ .

## NOTATION

$T_{\text{hot}}$	is the temperature of the hottest part of the lead;
$I$	is the current;
$I_{300}$	is the current at which the hottest part of the lead reaches room temperature;
$I_{\text{mp}}$	is the current producing a temperature equal to the melting point;
$S$	is the current-carrying cross section;
$\tau_{\text{wor}}$	is the magnet working time;
$\tau_{\text{sup}}$	is the magnet supply time.

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# THERMODYNAMIC PROPERTIES OF GeSe IN SOLID AND LIQUID STATES

S. M. Rasulov and R. A. Medzhidov

UDC 549.31

In view of the possible practical applications of germanium monoselenide (GeSe) and the occurrence of a type II phase transition in it, it was of interest to investigate its thermodynamic properties. The thermodynamic properties of GeSe in the temperature range 55-300°K have been investigated by V. M. Zhdanov. The literature contains no data for temperatures above 300°K.

In the present work the thermodynamic properties of GeSe in the range 0-820°C were investigated. The enthalpy of a specimen was experimentally measured by the method of mixtures.

The enthalpy data were used to construct interpolation equations, whose coefficients were determined by the least-squares method. The relationships between the thermodynamic functions and the enthalpy expressions were used to find equations for the specific heat, entropy, and thermodynamic potential.

The obtained data showed that the main theoretical relationship above the Debye temperature (Dulong and Petit law) becomes invalid. According to this law the specific heat at constant volume, per gram-atom, is a constant;  $3R \approx 6$  cal/g·atom·deg. Deviation of the specific heat of GeSe from the Dulong and Petit Law occurs above 20°C and increases with increase in temperature. The following possible reasons for this deviation are suggested.

1. The value of the specific heat at constant pressure ( $c_p$ ), and not of the specific heat at constant volume ( $c_v$ ), were found experimentally.
2. The anharmonicity of the vibrations makes a contribution to the specific heat.
3. The electron concentration is a sensitive function of temperature, and the energy required for the appearance of conduction electrons makes a substantial contribution to the specific heat. Free electrons, whose energy changes with temperature, have an insignificant effect on the specific heat.

At the temperature of the type II phase transition ( $t_{pt} = 620^\circ\text{C}$ ) and at the melting point ( $t_m = 670^\circ\text{C}$ ) there is an anomalous change in the thermodynamic properties.

The latent heat and entropy of melting were found and were  $2723 \pm 16$  cal/g-atom and  $2.890 \pm 0.017$  cal/g-atom·deg, respectively.

After melting the enthalpy increases linearly with increase in temperature, while the specific heat remains constant.

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# STEADY-STATE THERMAL CONDUCTIVITY OF A PLATE WITH A VARIABLE HEAT-TRANSFER COEFFICIENT

S. I. Prokopets and S. B. Tishechkin

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The study of heat-conduction problems involving a variable heat-transfer coefficient is now receiving great attention. In most of the known studies the heat-transfer coefficient varied with time or ambient temperature. There are far fewer studies in which the heat-transfer coefficient depends on the coordinate. Yet the last case is of theoretical and practical interest.

The solution of the following boundary-value problem is considered:

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} = 0 \quad (-\infty < x < +\infty, \quad 0 \leq y \leq h), \quad (1)$$

$$t = T_0 = \text{const at } y = 0, \quad (2)$$

$$\lambda \frac{\partial t}{\partial x} + \alpha(x)t = 0 \quad \text{at } y = h, \quad (3)$$

$$\frac{\partial t}{\partial x} \rightarrow 0 \quad \text{as } x \rightarrow \pm \infty. \quad (4)$$

Assuming that we can put

$$\alpha(x) = \alpha_0 + \varepsilon \alpha_1(x), \quad (5)$$

where

$$\alpha_1(x) = \begin{cases} A, & |x| \leq a, \\ 0, & |x| > a, \end{cases} \quad (6)$$

and  $\varepsilon$  is a small numerical parameter ( $\varepsilon \ll 1$ ), we seek the solution of problem (1)-(4) in powers of  $\varepsilon$  by the perturbation method:

$$t(\xi, \eta) = t_0(\eta) + \varepsilon t_1(\xi, \eta) + \varepsilon^2 t_2(\xi, \eta) + \dots \quad (7)$$

For  $\alpha(x) = \alpha_0$  the solution has the form

$$t_0(\eta) = T_0 - \text{Bi}_0 T_0 \eta (1 + \text{Bi}_0)^{-1} = f(\eta). \quad (8)$$

Applying the complex Fourier transform in  $\xi$  to the problem of the first approximation and using the theory of residues, we obtain

$$t_1(\xi, \eta) = 2AD \sum_{n=1}^{\infty} \frac{\sin(v_n \eta) \cdot \text{sh}(bv_n) \exp(v_n \xi)}{v_n [(1 + \text{Bi}_0) \cos v_n - v_n \sin v_n]} \quad (\xi < -b), \quad (9)$$

$$t_1(\xi, \eta) = 2AD \sum_{n=1}^{\infty} \frac{\sin(v_n \eta) \cdot \text{ch}(v_n \xi) \exp(-v_n b)}{v_n [v_n \sin v_n - (1 + \text{Bi}_0) \cos v_n]} - \frac{AD}{1 + \text{Bi}_0} \eta \quad (\xi < b), \quad (10)$$

$$t_1(\xi, \eta) = 2AD \sum_{n=1}^{\infty} \frac{\sin(v_n \eta) \cdot \text{ch}(v_n b) \exp(-v_n \xi)}{v_n [v_n \sin v_n - (1 + \text{Bi}_0) \cos v_n]} \quad (\xi > +b). \quad (11)$$

Here  $\xi = x/h$ ,  $\eta = y/h$ ,  $b = a/h$ ,  $D = f(1)$ , and  $v_n$  are the roots of the transcendental equation  $\tan v = -v \text{Bi}_0^{-1}$ .

The formulated problem (1)-(6) was also solved by electric simulation on an R-network analog computer. A comparison of the results obtained analytically and by electric simulation for a wide range of geometric and thermophysical parameters indicated a satisfactory agreement by the first approximation ( $\varepsilon = 0.093$ ) when 5-6 terms of series (9)-(11) were retained. The difference did not exceed 10%.

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NUMERICAL SOLUTION OF SYSTEMS OF VOLTERRA INTEGRAL EQUATIONS OF THE SECOND KIND ARISING IN PROBLEMS OF HEAT CONDUCTION THROUGH MULTILAYER MEDIA

A. M. Aizen and I. S. Redchits

UDC 536.21:518.0

The problem of unsteady heat conduction through multilayer media can be solved by reducing the original problem to systems of linear Volterra equations of the second kind of a special form.

An algorithm is presented for the numerical solution of such systems. In this case the kernels of the Volterra integral equations have singularities, becoming infinite at the right-hand ends of the integration intervals. Therefore, ordinary numerical methods of solution do not suffice.

The systems of Volterra integral equations of the second kind of the convolution type under consideration are solved by the method of finite sums based on the use of the modified Newton-Cotes quadrature formula, taking account of weight.

The integrals are split into elementary intervals and then the modified quadrature formula is used. Since the elementary intervals are small, their end points are taken as nodes. In this way the numerical solution of the system of integral equations is reduced to the solution of a system of linear algebraic equations which can be accomplished completely on computers of common types.

The procedure discussed is applied also to the solution of a single Volterra equation of the second kind obtained in the problem of unsteady linear heat conduction through a two-layer wall. A concrete example is solved.

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SOLUTION OF JOINING PROBLEM FOR MULTILAYER MEDIA BY USING LAPLACE TRANSFORMS

K. V. Lakusta and M. P. Lenyuk

UDC 517.946

We consider a finite rod with a thermally insulated lateral surface consisting of  $n$  different homogeneous materials with a continuous distribution of heat sources of density  $f_i(x, t)$ , an initial temperature  $\varphi_i(x)$ , an initial time rate of change of temperature  $\psi_i(x)$ , and temperatures on its ends  $z_1(t)$  and  $z_n(t)$  ( $i = \overline{1, n}$ ). Each material is characterized by its thermal conductivity  $\lambda_i$ , specific heat  $\bar{c}_i$ , density  $\gamma_i$ , and period of relaxation of thermal stresses  $\tau_i$ .

If we denote the temperature of each layer by  $U_i(x, t)$ , we obtain the following mathematical problem: in the domain

$$D = \bigcup_{i=1}^n D_i, \quad D_i = \{(t, x), \quad 0 < t < T, \quad h_{i-1} < x < h_i, \quad i = \overline{1, n}\},$$

find the bounded sufficiently smooth solution of the system of equations

$$b_i^2 \frac{\partial^2 U_i}{\partial t^2} + c_i^2 \frac{\partial U_i}{\partial t} - a_i^2 \frac{\partial^2 U_i}{\partial x^2} = f_i(x, t),$$

satisfying the initial conditions

$$U_i|_{t=0} = \varphi_i(x); \quad \left. \frac{\partial U_i}{\partial t} \right|_{t=0} = \psi_i(x) \quad (i = \overline{1, n}),$$

the boundary conditions

$$U_1|_{x=h_0} = z_1(t); \quad U_n|_{x=h_n} = z_n(t)$$



and the joining conditions

$$U_i|_{x=h_i} - U_{i+1}|_{x=h_i} = \alpha_i(t),$$

$$k_i \frac{\partial U_i}{\partial x} \Big|_{x=h_i} - k_{i+1} \frac{\partial U_{i+1}}{\partial x} \Big|_{x=h_i} = \beta_i(t), \quad (i = \overline{1, n-1}).$$

We have introduced the following notation:

$$a_i^2 = k_i, \quad c_i^2 = \bar{c}_i \gamma_i, \quad b_i^2 = \frac{\lambda_i}{w_i^2}, \quad \text{and} \quad w_i = \sqrt{\frac{\lambda_i}{\bar{c}_i \gamma_i \tau_i}}$$

is the rate of propagation of heat.

A procedure for solving the problem by taking Laplace transforms with respect to  $t$  is presented and applied to the study of the temperature distribution in two butted finite rods. The solution for an  $n$ -layer rod which is infinite in one or both directions is obtained from the solution of the joining problem for an  $n$ -layer finite rod by setting the controlling coefficients in the solution equal to zero and replacing the boundary conditions by

$$\lim_{x \rightarrow -\infty} U_1(t, x) = 0; \quad \lim_{x \rightarrow +\infty} U_n(t, x) = 0.$$

In the limit as  $b_i \rightarrow 0$  the ordinary parabolic temperature distribution is obtained and as  $c_i \rightarrow 0$ , a purely wave temperature distribution in an  $n$ -layer one-dimensional body.

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#### STRAIGHT-LINE METHOD IN THE DERIVATION OF APPROXIMATE ANALYTIC SOLUTIONS TO PROBLEMS IN THERMAL CONDUCTION

K. G. Omel'chenko and V. A. Shiparev

UDC 536.2.023

The straight-line method has been used to derive approximate analytical solutions for heat conduction by reducing a system of ordinary differential equations to simple recurrence relations; the conduction equation

$$\sigma \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t},$$

$$0 < x < \delta, \quad 0 < t \leq t_h \tag{1}$$

is replaced by the system

$$\frac{\partial^2 u_n}{\partial z^2} - u_n + u_{n-1} = 0, \quad 1 \leq n \leq m,$$

$$u_n = u(x, nh), \quad h = \frac{t_h}{m}, \quad z = \frac{x}{\sqrt{\sigma h}}. \tag{2}$$

The solution to (2) takes the form

$$u_n = - \int_0^z u_{n-1}(y) \operatorname{sh}(z-y) dy + C_1 \exp z + C_2 \exp(-z), \tag{3}$$

where  $u_0$  ( $n = 0$ ) is determined by the initial condition for (1), namely,  $u_0 = \text{const}$ .

A distinctive feature of (3) is that

$$u_{n-1} = \sum_0^{n-1} a_{h,n-1} z^k \exp z + b_{h,n-1} z^k \exp(-z),$$

$$u_n = \sum_0^n a_{k,n} z^k \exp z + b_{k,n} z^k \exp(-z). \quad (4)$$

Here the coefficients  $a_{k,n}$  and  $b_{k,n}$  are related by the recurrence relations

$$a_{k,n} = -\frac{1}{2k} a_{k-1,n-1} - \frac{k+1}{2} a_{k+1,n}, \quad (5)$$

$$b_{k,n} = \frac{1}{2k} b_{k-1,n-1} + \frac{k+1}{2} b_{k+1,n}, \quad 1 \leq k < n.$$

The boundary conditions for (1) are used to determine  $a_{0,n}$  and  $b_{0,n}$ ; boundary conditions of the following general form are considered:

$$\text{at } x=0, \quad z=0, \quad A_0(t) u + B_0(t) \frac{\partial u}{\partial z} + C_0(t) = 0,$$

$$x=\delta, \quad z=\delta_1, \quad A_1(t) u + B_1(t) \frac{\partial u}{\partial z} + C_1(t) = 0 \quad (6)$$

for the case of discontinuity in  $\sigma$  and a moving boundary.

In all these cases, the solutions are defined by (4) with (5) and the corresponding boundary conditions for  $a_{0,n}$ ,  $b_{0,n}$ ; this method can be used also with more complicated heat-conduction problems.

#### NOTATION

$u$  is the temperature;  
 $\sigma$  is the thermal diffusivity;  
 $t$  is the time;  
 $x$  is the linear coordinate;  
 $\delta$  is the layer thickness;  
 $t_k$  is the total time;  
 $h$  is the time step;  
 $n$  is the number of time step.

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#### SOME ASPECTS OF LIQUID - VAPOR MASS TRANSFER

É. V. Veitsman

UDC 532.6+536.71+539.6

A pure liquid in contact with its own vapor can be described by the following equation for the one-dimensional case [1] in terms of the mass flux  $J$  of material in the interface region:

$$J = a' \left[ F(x, t) - \frac{\partial [F(x, t)x]}{\Delta_x \partial t} - \left( \frac{\partial \mu}{\partial \rho} \right)_{T, \tau_{11}, \tau_{22}} \cdot \text{grad } \rho(x, t) \right]. \quad (1)$$

If (1) is to be used in practice, relationships of the form  $\rho = \rho(x, t)$  and  $\psi = \psi(x, t)$ ;  $-\psi'_x = F(x, t)$  must be known.

These functions appear in a system of equations that describe the microscopic forces in the interface:

$$\left. \begin{aligned} \frac{\partial \rho}{\partial t} &= \frac{\partial}{\partial x} \left( \frac{\partial \psi}{\partial x} \cdot \frac{\rho}{\gamma} + D \cdot \frac{\partial \rho}{\partial x} \right), \quad (a) \\ \frac{\partial^2 \psi}{\partial x^2} - \frac{\psi}{\lambda^2} &= 4\pi k \rho(x, t), \quad (b) \end{aligned} \right\} \quad (2)$$

Instead of (2b) one can use an equation of the following form:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial \psi}{\lambda \partial x} = 4\pi k \rho(x, t) \cdot \exp\left(\mp \frac{x}{\lambda}\right).$$

It is very difficult to solve (2), but the difficulties can be overcome for the steady state by using the condition  $\delta\sigma = 0$ , i.e., the first variation of the surface tension is put as zero. Then the function  $\rho = \rho(x)$  takes the form given in [2], while  $\psi(x, t) = -4\pi k\lambda^2 \rho(x)$ ; if the deviation from the stationary state is small, then  $\psi(x, t) = -4\pi k\lambda^2 \rho(x, t)$ .

The expression for  $\bar{\mu}^+$  for the material in the interface region takes the form  $\bar{\mu}^+ = \mu_0^+ + RT \ln f(\rho) \cdot \rho$ :

$$f(\rho) = \frac{C}{\rho} \exp(\alpha\rho); \quad \alpha = 4\pi k\lambda^2/RT; \quad \rho_2 \exp(\alpha\rho_1) = \rho_1 \exp(\alpha\rho_2). \quad (3)$$

The function  $f(\rho)$  is called the interphase activity coefficient. It characterizes the effects of macroscopic forces in the interface region on  $\bar{\mu}^+$ .

Theoretical studies on mass transport in this region have provided the following results: 1) the equation of state for the material in the interface region (for the following systems: pure liquid-vapor, pure solid-vapor, equilibrium and nonequilibrium stationary states); 2) an expression for the energy of the intermolecular (interatomic) interactions in the liquid  $\psi_1$  and gas  $\psi_2$ :

$$\Delta L = 8\pi^2\sigma/RT (\rho_2 - \rho_1) \ln(\rho_2/\rho_1), \quad (4)$$

$$\psi_{1(2)} = -\frac{RT \ln(\rho_2/\rho_1)}{\rho_2 - \rho_1} \cdot \rho_{1(2)}. \quad (5)$$

Expressions (4) and (5) are readily verified; the values of  $\Delta L$  for  $T$  appreciably less than the critical temperature  $T_{cr}$  lie in the range from  $10^{-6}$  to  $10^{-7}$  cm for many different substances (from liquid helium to liquid platinum), while  $\Delta L$  tends to increase with the complexity of the molecular structure. Also,  $\Delta L \rightarrow 0$  for  $T \rightarrow T_{cr}$ .

The value of  $\psi_1$  is in the region of several kcal/mole for various liquids ( $T \ll T_{cr}$ ); at  $T = T_{cr}$ , (5) becomes  $\psi = -RT$  (after resolving the indeterminacy).

The latter formula readily gives a quantity  $a$ , which at  $T_{cr}$  is analogous to the quantity appearing in the van der Waals equation. The van der Waals constant is  $a = 9/8RT_{cr}v_{cr}$ , while the new quantity at  $T_{cr}$  takes the value  $a_{cr} = RT_{cr}v_{cr}$ .

#### NOTATION

$a'$	is the phenomenological coefficient;
$\rho$	is the density;
$\rho_1, \rho_2$	are the densities of liquid and gas;
$\bar{\mu}^+$	is the specific chemical potential;
$\tau_{11}$ and $\tau_{12}$	are the components of the stress tensor for the interface region (normal and tangential);
$\Delta x = v - v_x$	
$v$	is the speed of material at point $x$ ;
$v_x$	is the characteristic speed;
$\psi$	is the potential (J/kg) due to the intermolecular forces in the interface region;
$D$	is the mass-transport coefficient;
$\lambda$	is the screening coefficient (m);
$k$	is the constant ( $m^3/kg \cdot sec^2$ );
$C$	is the constant of integration;
$v_{cr}$	is the specific volume of substance at $T_{cr}$ .

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EFFECT OF TEMPERATURE DEPENDENCE OF RESISTIVITY OF A CYLINDRICAL CONDUCTOR HEATED BY A CURRENT ON THE TEMPERATURE, CURRENT, AND ELECTRIC FIELD DISTRIBUTIONS

R. S. Kuznetskii

UDC 538.56

The steady-state distributions of temperature and electric field in a cylindrical conductor  $r \leq 1$  of radius  $r_0$  heated by a current of frequency  $\omega$ , with the electric vector at the surface  $e_0$  collinear with the generators of the cylinder, having a thermal conductivity  $\lambda$ , an absolute magnetic permeability  $\mu$ , a temperature coefficient of resistivity  $\alpha$ , and with surface heat transfer, are described by the system of nonlinear differential equations

$$\rho(t'' + r^{-1}t') = -e^2, \quad \rho(e'' + r^{-1}e') = in^2e; \quad t(1) = t'(0) = e'(0) = 0, \quad e(1) = 1, \quad (1)$$

where  $j = e/\rho$  and the temperature dependence of  $\rho$  is specified as  $\rho(t) = kt + 1$ . Here  $r$  is the running radius-vector;  $t$  is the temperature of the conductor measured from the temperature of its surface;  $\rho = \rho(t)$  is its resistivity;  $\rho_0$  is its dimensional resistivity at  $t = 0$ ;  $e \equiv \varepsilon \exp(i\varphi)$  and  $j = \iota \exp(i\varphi)$  are the complex amplitudes of the electric field and the current density ( $\iota = \varepsilon/\rho$ ), relative, respectively, to the quantities  $r_0$ ,  $(e_0 r_0)^2 / [\lambda \rho_0 (2 - \delta_n)]$ ,  $\rho_0$ ,  $e_0$ , and  $e_0/\rho_0$ ;  $n \equiv r_0 \sqrt{\mu \omega / \rho_0}$  is the frequency criterion; and  $k \equiv [(e_0 r_0)^2 / \lambda \rho_0] [\alpha / (2 - \delta_n)]$  is the nonlinearity criterion;  $\delta_n \equiv 1$  if  $n = 0$ , and  $\varphi_n \equiv 0$  if  $n \neq 0$ .

We are interested in the following functionals, which depend on the criteria  $n$  and  $k$ , for functions  $t(r)$  and  $e(r)$  defined by Eqs. (1), the maximum  $t(0)$  and the average temperature of the conductor  $\langle t \rangle = \int_0^1 t dr^2$ , which characterize the nonuniformity of the heating; the total relative change in resistivity  $\delta \rho \equiv \rho[t(0)] - 1 = kt(0)$ , which is absent when  $k = 0$ ;  $\varepsilon(0)$  and  $\iota(0)$ , characterizing the field and current skin effects, which are identical for  $k = 0$ ; and the phase difference  $\varphi(0)$  between the electric field and current density on the axis and on the boundary.

The dependence of the quantities under study on the controlling criteria  $k$  and  $n$ , shown graphically in Fig. 1 for a cylindrical conductor, is qualitatively the same as for a flat conductor. The behavior of a cylindrical conductor differs quantitatively from that of a flat conductor with the same  $k$  and  $n$  in having appreciably smaller values of  $t(0)$ ,  $\langle t \rangle$ , and  $\delta \rho$ , larger values of  $\varepsilon(0)$ ,  $\iota(0)$ , and  $\varphi(0)$ , a considerably greater difference between  $\langle t \rangle$  and  $t(0)$ , and a weaker dependence of all these quantities on  $n$ .

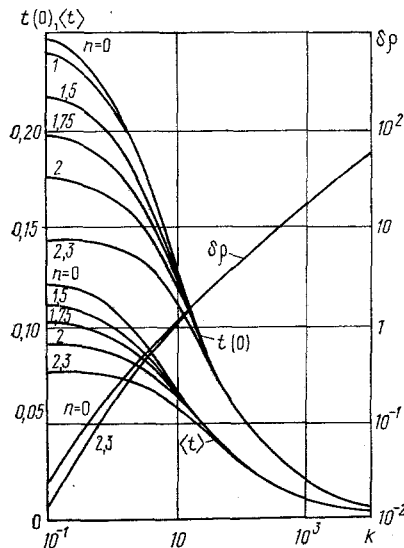


Fig. 1. Maximum (on the axis  $r = 0$ )  $t(0)$  and average temperature  $\langle t \rangle$  of a cylindrical conductor and the total relative change of its resistivity  $\delta \rho$  as functions of the nonlinearity criterion  $k$  for various values of the frequency criterion  $n$ .

A theoretical analysis of the solutions of Eqs. (1) for  $k \gg \max(1; n^4)$  leads to the following asymptotic relations:

$$\begin{aligned} t(0) &\approx \frac{0.58150}{\sqrt{k}}, \quad \langle t \rangle \approx \frac{0.3360}{\sqrt{k}}, \quad \frac{\langle t \rangle}{t(0)} \approx 0.5778, \quad \delta\rho \approx 0.58150\sqrt{k}, \\ \varepsilon(0) &\approx 1 - 0.080929 \frac{n^4}{k} \approx 1, \quad \iota(0) \approx \frac{1.6978}{\sqrt{k}}, \quad \varphi(0) \approx -0.58801 \frac{n^2}{\sqrt{k}}. \end{aligned} \quad (2)$$

These relations have the same general structure as those for a flat conductor, but with somewhat different constants. For larger values of  $k$  the quantities  $t, \rho, \iota, \varphi$ , and practically also  $\varepsilon \approx 1$  become independent of the frequency criterion  $n$ . In this case  $t, \iota$ , and  $\varphi$  are very small (the field and current are nearly real), but  $\rho$  is large; these statements apply to  $\iota$  and  $\rho$  outside a surface layer of the cylinder of thickness of the order  $(k \ln k)^{-1/2}$ .

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TEMPERATURE, HEAT FLUX, AND ELECTROMAGNETIC  
FIELD DISTRIBUTIONS IN A CYLINDRICAL CONDUCTOR  
WITH A TEMPERATURE-DEPENDENT CONDUCTIVITY AT  
LOW FREQUENCIES

R. S. Kuznetskii

UDC 538.56

In a cylindrical conductor  $r \leq 1$  of radius  $r_0$  with surface cooling, the steady-state temperature and electromagnetic field distributions are described by the system of nonlinear differential equations

$$Dt = -\sigma(u^2 + v^2), \quad Du = -n^2\sigma v, \quad Dv = n^2\sigma u \left( D = \frac{1}{r} \frac{d}{dr} r \frac{d}{dr} \right), \quad (1)$$

where  $h = -in^{-2}e'$ , with the following boundary conditions:  $u(1) = 1, v(1) = t(1) = u'(0) = v'(0) = t'(0) = 0$ . Here  $r$  is the running radius-vector,  $t$  is the temperature of the conductor measured from the temperature of its surface,  $\sigma = \sigma(t)$  is its conductivity,  $\sigma_0$  is the dimensional conductivity at  $t = 0, q = -t'$  is the heat flux density,  $e \equiv u + iv$  is the complex amplitude of the electric field,  $e_0$  is the dimensional amplitude at the surface, and  $h = n^{-2}(v' - iu')$  is the complex amplitude of the magnetic field; these quantities are made dimensionless by dividing, respectively, by  $r_0, (\sigma_0/\lambda)[(e_0 r_0)^2/(2 - \delta_n)], \sigma_0 e_0^2 r_0/(2 - \delta_n), e_0$ , and  $\sigma_0 e_0 r_0; n = r_0 \sqrt{\sigma_0 \mu \omega}$  is the frequency criterion,  $\omega$  is the angular frequency of the current,  $\lambda$  and  $\mu$  are the thermal conductivity and absolute magnetic permeability of the conductor;  $\delta_n \equiv 1$  if  $n = 0$  and  $\delta_n \equiv 0$  if  $n \neq 0$ . From functionals of problem (1) are found the effective resistance  $R = (n^2/2\sigma_0 s) \langle v'/|e'|^2 \rangle_{r=1}$  and the self-inductance  $L = (\mu/2\pi) \langle u'/|e'|^2 \rangle_{r=1}$  per unit length of the conductor ( $s \equiv \pi r_0^2$ ). We examine the behavior of the functions  $t(r), q(r), e(r), h(r)$ , and functionals in the limit  $n \ll 1$  (and  $n = 0$ ), which turns out to be nontrivial for  $\sigma' \neq 0$ .

For  $n = 0$  we have  $e = 1$ , and we use a superscript 0 to denote the values of  $t(r)$  and  $q(r)$  given by Eq. (1),  $Dt + \sigma = 0$  for  $t(1) = t'(0) = 0$ . For nonzero  $n \ll 1$  we find for  $t, q, |e|, \varphi \equiv \arg e, |h|$ , and  $\psi \equiv \arg h$ :

$$t \approx t^0 + n^4 \vartheta, \quad q \approx q^0 - n^4 \vartheta'; \quad u \approx 1 - n^4 \vartheta, \quad v \approx -n^2 \left( t^0 + n^4 \int_r^1 J \cdot dr \right); \quad (2)$$

$$e \approx 1 - n^4 y + in^2 t^0, \quad |e| \approx 1 + n^4 \left[ y - \frac{1}{2} (t^0)^2 \right] \approx 1, \quad \varphi \approx -n^2 t^0 < 0, \quad |\varphi| \ll 1; \quad (3)$$

$$h \approx q^0 + n^4 J + in^2 y', \quad |h| \approx q^0 \equiv n^4 \left[ J + \frac{(y')^2}{2q^0} \right] \approx q^0, \quad \psi \approx -n^2 \frac{y'}{q^0} < 0, \quad |\psi| \ll 1, \quad (4)$$

where  $q^0(r) = (-t^0)'$ ,  $\sigma^0 \equiv \sigma(t^0), J(r) \equiv \int_0^r [(\sigma^0)' \vartheta - \sigma^0 y] r dr$ , and  $y(r), t(r)$  are solutions of the equations

$$Dy = t^0 \sigma^0, \quad y(1) = y'(0) = 0 \quad \text{и} \quad D\vartheta + (\sigma^0)' \vartheta = \sigma^0 [2y - (t^0)^2], \quad \vartheta(1) = \vartheta'(0) = 0. \quad (5)$$

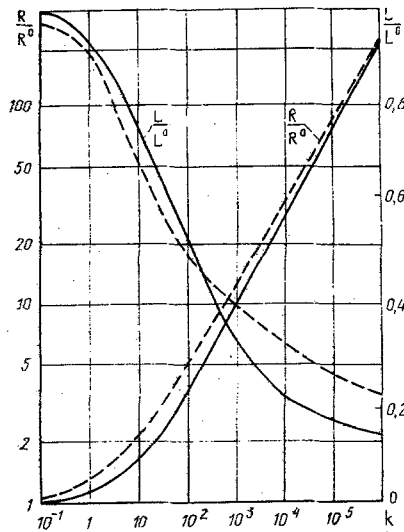


Fig. 1. Dimensionless effective resistance  $R/R^0$  and self-inductance  $L/L^0$  of a cylindrical conductor as functions of the nonlinearity criterion  $k$  at low frequencies (small values of the frequency criterion  $n$ ). Both dimensionless quantities are unity for  $k = n = 0$ . The open curves give the analogous relations for a flat conductor.

We have for  $R$  and  $L$ , respectively,

$$2 \sigma_0 s \cdot R \approx \frac{1}{q^0(1)} \left\{ 1 - \frac{n^4}{[q^0(1)]^2} [[y'(1)]^2 + q^0(1) J(1)] \right\} \approx \frac{1}{q^0(1)}, \quad (6)$$

$$\frac{2\pi}{\mu} L \approx -\frac{y'(1)}{[q^0(1)]^2} \left\{ 1 - \frac{n^4}{[q^0(1)]^2} [[y'(1)]^2 + 2q^0(1) J(1)] \right\} \approx -\frac{y'(1)}{[q^0(1)]^2}. \quad (7)$$

In all the expressions obtained, and in Fig. 1, we take  $\sigma(t) = (kt + 1)^{-1}$ , where  $k \equiv (e_0 r_0)^2 (\sigma_0 / \alpha) [\alpha / (2 - \delta_n)]$  is the nonlinearity criterion of the problem and  $\alpha$  is the temperature coefficient of resistivity. In particular, for  $k \gg 1$  we obtain the asymptotic relations

$$e \approx 1, \quad t(0) \approx \frac{0.58150}{\sqrt{k}}, \quad q(1) \approx \sqrt{\frac{\ln k}{k}} \approx h(1); \quad (8)$$

$$R \approx R^0 \sqrt{\frac{k}{\ln k}}, \quad L \approx L^0 \frac{2}{\ln k}, \quad (9)$$

where  $R^0 \equiv (\sigma_0 s)^{-1}$  and  $L \equiv (8\pi)^{-1} \mu$  correspond to  $k = n = 0$ . For the same  $k$ , the value of  $L/L^0$  is two thirds as large as for a flat conductor. Asymptotic expressions for quantities of the type (8), (9) hold independently of the value of  $n$  as a consequence of their well-known self-similarity with respect to the criterion  $n$  for  $k \gg \max(1; n^4)$ .

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MEASUREMENT AND SIMULATION OF THE  
MELTING OF SCRAP STEEL IN  
MOLTEN IRON

M. Ya. Medzhibozhskii, S. M. Grigorenko,  
L. I. Khiish, A. A. Lykin,  
and L. V. Rebrov

UDC 669.18:621.745.55

The experiments were performed with a laboratory induction frequency of capacity 20 kg with natural or forced convection. Cylindrical steel specimens of diameter 10 mm and length 100 mm were immersed for a set period in the molten steel, which had a constant composition, constant temperature, and steady air flow rate. The carbon contents were 2-3.75% in the various runs, while the temperatures were -1400-1600°C and the gas release rates were -0-1 m<sup>3</sup>/m<sup>2</sup>.sec.

It is found that the dissolution of steel group scrap occurs in two periods and four stages: period I is the thermal one and includes the stages of melt freezing on the surface of the group, melting of the frozen layer, and heating of the specimen; period II is that of rapid dissolution of the scrap. During the first stage, the rate of penetration of heat into the specimen exceeds the rate of supply of heat from the melt, which results in layer being frozen on the surface. During the second stage, the external heat flux exceeds the internal flux, so the frozen layer melts again. In the third stage, the concentration in the surface layer falls suddenly [%C<sub>sur</sub>], which causes carbon to diffuse from the melt into the specimen, the surface layer becoming carburized. The scrap melts rapidly in the fourth stage.

It has been found that it is sufficient to carburize a very thin surface layer in order to cause rapid melting; the nonstationary diffusion problem has been solved to show that the carburization rate and the melting rate for the surface layers are not limited by the melting rate as a whole, since the latter is restricted by the heat and mass transfer in the liquid phase in stage 4.

These results have been used in a mathematical model for the coefficients of all stages; calculations on the first two stages involve solving finite-difference equations for the nonstationary thermal conduction subject to boundary conditions of the first kind in the presence of specimens varying in radius. Calculations for the third stage involve nonstationary thermal conduction with boundary conditions of the third kind. The final surface temperature of the specimen (t<sub>sur</sub>) has been determined by solving the equations for heat and mass transport (the latter for carbon) on the basis of the relation between the liquidus temperature and the carbon concentration:

$$V_{\kappa} = \frac{\alpha (t_l - t_{sur})}{\rho_{scr} [\eta_{lat} + C_{scr} (t_l - t_{sur})]} = \frac{\beta_c \{ [\%C]_l - [\%C]_{sur} \} \rho_l}{\{ [\%C]_l - [\%C]_{scr} \} \rho_{scr}}, \quad (1)$$

$$t_{sur} = 1536 - 54 [\%C]_{sur} - 9.13 [\%C]_{sur}^2, \quad (2)$$

where  $\alpha$  is the heat-transfer coefficient,  $\beta_c$  is the coefficient for carbon transfer from the melt to the surface of the scrap,  $t_l$  and  $t_{sur}$  are the temperature of the liquid and surface of the specimen, [%C] is the carbon concentration.

The duration of the fourth stage is defined by  $\tau = r/V_{\kappa}$ , where  $V_{\kappa}$  is the linear dissolution rate given by (1). Calculations on the dissolution time for steel cylinders of diameter 200 mm give values close to those actually found for Martens and converter modes of steel production.

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On the basis of an analysis of an approximate temperature solution it is shown that in the regular stage under identical conditions of heat exchange at a surface the temperature curves for walls of different curvature coincide rather closely at certain times. This property is called the invariance of the shape of temperature curves. The times at which this coincidence of the curves occurs are called the equivalent times.

Analytical functions are obtained which determine the equivalent times for walls of different curvature, both surfaces of which participate in heat exchange with mixed boundary conditions of the second and third kinds.

It is shown that the property of invariance is confirmed by direct calculations using a strict analytical solution.

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EFFECT OF UNSTEADY HEAT CONDUCTION ON THE  
RESULTS OF THE MEASUREMENT OF TEMPERATURE  
AND LOCAL HEAT FLUXES WITH A  
DYNAMIC THERMOCOUPLE

S. P. Polyakov and P. F. Bulanyi

UDC 537.562.083

One of the largest errors in a measurement with the thermocouples is due to the drainage of heat from the injection into the thermocouple leads [1]. The principle of the measurement of heat fluxes is based on the recording of the rate of heating of the thermocouple and the subsequent differentiation of the heating curve. To calculate the departing heat flux one solves the equation of unsteady heat conduction by the method of the Laplace transform [2] with null initial conditions and boundary conditions of the second kind:

$$T(x, 0) = 0; \quad T(0, t) = T_j [1 - \exp(-kt)]; \quad T(\infty, t) = 0.$$

The solution obtained has the following form:

$$T(x, t) = T_j \Phi^* \left\{ \frac{x}{2\sqrt{at}} \right\} - \frac{T_j}{2} \exp(-kt) \left[ \exp \left( -ix \sqrt{\frac{k}{a}} \right) \Phi^* \left\{ \frac{x}{2\sqrt{at}} - i\sqrt{kt} \right\} + \exp \left( ix \sqrt{\frac{k}{a}} \right) \Phi^* \left\{ \frac{x}{2\sqrt{at}} + i\sqrt{kt} \right\} \right],$$

where

$$\Phi^* \left\{ \frac{x}{2\sqrt{at}} \right\} = \frac{2}{\sqrt{\pi}} \int_{\frac{x}{2\sqrt{at}}}^{\infty} \exp(-y^2) dy.$$

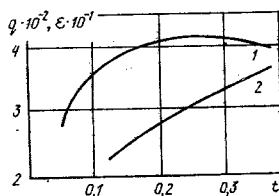


Fig. 1. Heat flux into thermocouple leads (curve 1) and relative error in measurement of heat fluxes (curve 2).  $q$ , cal/cm·sec;  $\epsilon$ , %;  $t$ , sec.



The heat flux into the thermocouple leads is calculated by the Fourier law through differentiation of the temperature distribution obtained. From the equation for the variation in heat content of the junction and the equation for the departing flux one obtains the temperature reduction and the local heat fluxes at any time.

The results obtained are tested experimentally using thermocouples of different constructions. Unsteady heating is accomplished in a high-temperature gas stream discharging from a plasmotron. The appropriate calculation (Fig. 1) is made for the data taken from the experiment, with the values  $k = 3.26 \text{ sec}^{-1}$ ,  $T = 2270^\circ\text{K}$ ,  $\alpha = 0.055 \text{ cal/cm}^2 \cdot \text{sec} \cdot \text{deg}$ , and  $d = 0.5 \text{ mm}$ . The experimental results obtained agree with the proposed theoretical equations for calculating the measurement errors.

#### NOTATION

- $\alpha$  is the thermal diffusivity of thermocouple material;
- $k$  is the reciprocal of thermocouple time constant;
- $\alpha$  is the heat-transfer coefficient;
- $T_j$  is the temperature of plasma jet;
- $t$  is the time.

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Original article submitted November 14, 1975.

### THEORETICAL STUDY OF THE QUESTION OF USING THE ELECTORHEOLOGICAL EFFECT IN REGENERATIVE HEAT-EXCHANGE APPARATUS

V. K. Gleb

UDC 536.27

The case of heat exchange through a plane partition (Fig. 1a and b) is discussed.

An electrorheological suspension (a viscoplastic heat-transfer agent, subscript 1 and single superscript prime) moves on one side of the barrier and a Newtonian liquid moves on the other. We assume that the outer surfaces of the heat exchanger are adiabatic and we neglect heat transfer in the partition in the axial direction. We will take as given and constant the densities  $\rho_1$  and  $\rho_2$ , heat capacities  $c_1$  and  $c_2$ , and heat-exchange coefficients  $\alpha_1$  and  $\alpha_2$  on each side of the partition which has a known thermal conductivity  $\lambda_c$ . Under these assumptions the mathematical formulation of the problem comes down to the system of equations

$$\begin{cases} \frac{\alpha_1}{H_1 \rho_1} (\theta' - t') = \omega_1 \frac{dc_1 t'}{dx} & \text{(zone I),} \\ \frac{\alpha_2}{H_2 \rho_2} (\theta'' - t'') = \omega_2 \frac{dc_2 t''}{dx} & \text{(zone II),} \\ \frac{d^2 \theta}{dz^2} = 0 & \text{(partition),} \end{cases} \quad (1) \quad (2) \quad (3)$$

$$\omega_1 = \omega_1(E, \mu_p, \Delta P, m, n), \quad \omega_2 = \frac{\Delta P_2 H_2^2}{12 \mu} = \text{const.}$$

The boundary conditions can be represented in the form

$$\begin{aligned} -\lambda_c \frac{d\theta}{dz} \Big|_{z=-\frac{\delta}{2}} &= \alpha_2 (t'' - \theta''), \\ -\lambda_c \frac{d\theta}{dz} \Big|_{z=\frac{\delta}{2}} &= \alpha_1 (\theta' - t'). \end{aligned}$$

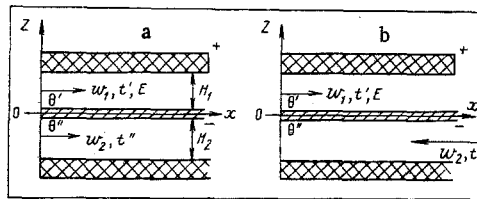


Fig. 1. Diagram of a heat exchanger containing a plane partition with parallel flow of heat-transfer agents (a: direct flow; b: counterflow).

Keeping in mind the quadratic dependence of the initial shear stress ( $\tau_0 = \alpha E^2$ ) [1] and the influence of the electrorheological effect on the velocity profile [2], as a result of the solution of the initial system of equations we obtain a characteristic curve expressing the effect of an electric field on the efficiency of regenerative heat exchange.

For direct flow

$$\frac{t' - t'_1}{t' - t'_1} = 1 + K_1 E^2 \frac{3 \alpha L}{\Delta P_1 H_1}.$$

for counterflow

$$\frac{t' - t''_1}{t' - t''_1} = 1 + K_2 E^2 \frac{3 \alpha L}{\Delta P_1 H_1}.$$

The parameters  $K_1$  and  $K_2$  allow for the geometrical dimensions of the channel and the rheological properties of the heat-transfer agents. We note that with an increase in the electric field strength the equalization of temperatures of the heat-transfer agents occurs in a smaller distance, which improves the heat-exchange characteristics of the apparatus and reduces the energy expenditure.

#### NOTATION

$\theta', \theta''$	are the temperatures of partition walls;
$\theta$	is the variable temperature of partition;
$t'_1, t''_1$	are the initial temperatures;
$\delta$	is the partition thickness;
$L$	is the length of heat exchanger;
$E$	is the electric strength;
$m, n$	are the rheological parameters;
$\alpha$	is the proportionality factor;
$\Delta P_1, \Delta P_2$	are the pressure drops.

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